1. **Define the Bayesian interpretation of probability.**
2. The Bayesian interpretation of probability is a philosophical and statistical viewpoint that views probability as a measure of uncertainty or degree of belief in an event, given the available evidence or information. In the Bayesian framework, probability is assigned subjectively based on prior knowledge, observations, and data, and it is updated as new evidence becomes available.

Key principals of **Bayesian interpretation of probability are :**

Subjective Probability

Prior and Posterior Probability

Bayes' Theorem

Iterative Process

Bayesian Inference

1. **Define probability of a union of two events with equation.**

The probability of the union of two events A and B, denoted as P(A ∪ B), represents the probability that at least one of the events A or B occurs. In other words, it is the probability that either event A occurs, or event B occurs, or both events A and B occur.

The equation for the probability of the union of two events is given by the addition rule of probability:

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

Where:

P(A) is the probability of event A.

P(B) is the probability of event B.

P(A ∩ B) is the probability of the intersection of events A and B, i.e., the probability that both events A and B occur.

1. **What is joint probability? What is its formula?**

Joint probability is a concept in probability theory that represents the probability of two or more events occurring simultaneously. It quantifies the likelihood of the intersection of multiple events happening together.

For two events A and B, the joint probability is denoted as P(A and B) or P(A ∩ B), where "∩" represents the intersection of the two events.

The formula for calculating the joint probability of two events A and B is given by the multiplication rule of probability:

P(A ∩ B) = P(A) \* P(B|A)

Where:

P(A ∩ B) is the joint probability of events A and B occurring simultaneously.

P(A) is the probability of event A happening.

P(B|A) is the conditional probability of event B occurring given that event A has already occurred.

1. **What is chain rule of probability?**

The chain rule of probability is a fundamental principle in probability theory that allows us to decompose the joint probability of multiple events into conditional probabilities. It is a way to express the probability of a sequence of events as a product of conditional probabilities.

For a sequence of events A₁, A₂, A₃, ..., Aₙ, the chain rule of probability can be written as:

P(A₁ ∩ A₂ ∩ A₃ ∩ ... ∩ Aₙ) = P(A₁) \* P(A₂|A₁) \* P(A₃|A₁ ∩ A₂) \* ... \* P(Aₙ|A₁ ∩ A₂ ∩ A₃ ∩ ... ∩ Aₙ₋₁)

Where:

* P(A₁ ∩ A₂ ∩ A₃ ∩ ... ∩ Aₙ) represents the joint probability of all n events occurring simultaneously.
* P(A₁) is the probability of event A₁ happening.
* P(A₂|A₁) is the conditional probability of event A₂ occurring given that event A₁ has already occurred.
* P(A₃|A₁ ∩ A₂) is the conditional probability of event A₃ occurring given that events A₁ and A₂ have already occurred.
* And so on, up to P(Aₙ|A₁ ∩ A₂ ∩ A₃ ∩ ... ∩ Aₙ₋₁), which is the conditional probability of event Aₙ occurring given that events A₁, A₂, A₃, ..., Aₙ₋₁ have already occurred.

1. **What is conditional probability means? What is the formula of it?**

Conditional probability is a concept in probability theory that represents the probability of an event A occurring given that another event B has already occurred. It quantifies the likelihood of event A happening under the condition or constraint that event B is known to be true.

The conditional probability of event A given event B is denoted as P(A|B), read as "the probability of A given B." It can be calculated using the following formula:

P(A|B) = P(A ∩ B) / P(B)

Where:

P(A|B) is the conditional probability of event A given event B.

P(A ∩ B) represents the joint probability of events A and B occurring simultaneously.

P(B) is the probability of event B happening.

1. **What are continuous random variables?**
2. Continuous random variables are variables in probability theory and statistics that can take on any value within a specified range or interval. Unlike discrete random variables, which can only assume specific isolated values, continuous random variables have an infinite number of possible values within a continuous range.
3. **What are Bernoulli distributions? What is the formula of it?**
4. he Bernoulli distribution is a discrete probability distribution that models a random experiment with two possible outcomes: success (usually denoted as "1") or failure (usually denoted as "0"). It is often used to represent a single trial with a binary outcome, where the probability of success is denoted by "p," and the probability of failure (1-p) is complementary.

The probability mass function (PMF) of the Bernoulli distribution is given by the following formula:

P(X = x) = p^x \* (1-p)^(1-x)

Where:

* P(X = x) is the probability of the random variable X taking the value x (either 0 or 1).
* "p" is the probability of success (probability of X being 1).
* (1-p) is the probability of failure (probability of X being 0).
* x is the outcome of a single trial (either 0 or 1).

1. **What is binomial distribution? What is the formula?**
2. The binomial distribution is a discrete probability distribution that models the number of successes in a fixed number of independent Bernoulli trials. A Bernoulli trial is an experiment with two possible outcomes: success (usually denoted as "1") or failure (usually denoted as "0"). In a binomial distribution, each trial is independent, and the probability of success "p" remains constant across all trials.

The probability mass function (PMF) of the binomial distribution is given by the following formula:

P(X = k) = C(n, k) \* p^k \* (1-p)^(n-k)

Where:

* P(X = k) is the probability of getting exactly "k" successes in "n" trials.
* "C(n, k)" represents the binomial coefficient, also known as "n choose k," and is calculated as C(n, k) = n! / (k! \* (n-k)!), where "!" denotes factorial.
* "p" is the probability of success in a single trial.
* (1-p) is the probability of failure in a single trial.
* "k" is the number of successes we are interested in counting.

1. **What is Poisson distribution? What is the formula?**
2. The Poisson distribution is a discrete probability distribution that models the number of events that occur within a fixed interval of time or space, given a known average rate of occurrence. It is often used to describe rare or infrequent events, where the probability of multiple occurrences within a short time period is small.

The probability mass function (PMF) of the Poisson distribution is given by the following formula:

P(X = k) = (e^(-λ) \* λ^k) / k!

Where:

* P(X = k) is the probability of observing "k" events within the given interval.
* "e" is the base of the natural logarithm, approximately equal to 2.71828.
* "λ" (lambda) is the average rate of events per interval (mean number of occurrences).
* "k" is the number of events we are interested in counting.
* k! represents the factorial of k.

1. **Define covariance.**

Covariance is a statistical measure that quantifies the degree to which two random variables change together. It indicates the relationship between two variables, showing whether they tend to increase or decrease together, or if they have no linear relationship.

For a pair of random variables X and Y, the covariance is denoted as Cov(X, Y) and is calculated using the following formula:

Cov(X, Y) = Σ [(Xᵢ - μₓ) \* (Yᵢ - μᵧ)] / n

Where:

Xᵢ and Yᵢ are individual observations of the random variables X and Y, respectively.

μₓ is the mean (average) of the random variable X.

μᵧ is the mean (average) of the random variable Y.

n is the number of observations in the dataset.

1. **Define correlation**

Correlation is a statistical measure that quantifies the strength and direction of the linear relationship between two or more variables. It is used to understand how changes in one variable are associated with changes in another variable. Correlation ranges from -1 to 1, where -1 indicates a perfect negative correlation (inverse relationship), 1 indicates a perfect positive correlation (direct relationship), and 0 indicates no linear relationship (variables are not correlated).

The two most commonly used measures of correlation are Pearson correlation coefficient and Spearman rank correlation coefficient

1. Pearson Correlation Coefficient: The Pearson correlation coefficient, also known as Pearson's r, measures the linear relationship between two continuous variables. It is calculated as the covariance of the two variables divided by the product of their standard deviations:

r = Cov(X, Y) / (σₓ \* σᵧ)

Where:

* Cov(X, Y) is the covariance between variables X and Y.
* σₓ and σᵧ are the standard deviations of variables X and Y, respectively.

1. Spearman Rank Correlation Coefficient: The Spearman rank correlation coefficient assesses the strength of a monotonic relationship between two variables. A monotonic relationship means that as one variable increases, the other variable either increases or decreases but not necessarily at a constant rate. The Spearman rank correlation is calculated based on the ranks of the data rather than the original values, making it suitable for both continuous and ordinal variables.

The Spearman rank correlation coefficient ranges from -1 to 1, where -1 indicates a perfect negative monotonic relationship, 1 indicates a perfect positive monotonic relationship, and 0 indicates no monotonic relationship.

1. **Define sampling with replacement. Give example.**
2. Sampling with replacement is a sampling technique in statistics where each element in a population or dataset has an equal chance of being selected at each draw, and after being selected, it is placed back into the population before the next draw. This means that the same element can be selected multiple times during the sampling process.

**Example** :

Suppose we have a bag containing four colored balls: red, blue, green, and yellow. We want to sample three balls from the bag using the sampling with replacement technique.

Step 1: We reach into the bag and randomly select one ball. Let's say we draw a red ball. After noting down the color, we place the red ball back into the bag, ensuring that the bag always contains four balls.

Step 2: We reach into the bag again and randomly select another ball. This time, we draw a green ball. We note down the color, put the green ball back into the bag, and now the bag contains four balls again.

Step 3: We perform the final draw. We reach into the bag one more time and randomly select a ball. This time, we draw a blue ball. We note down the color and put the blue ball back into the bag.

After three draws, we have the following sequence of colors: red, green, blue. Each draw was made with replacement, meaning that the same ball color could be chosen multiple times.

In sampling with replacement, the probability of drawing any color in each step remains 1/4 (assuming all balls have an equal chance of being drawn), as the selection process is independent and identical at each draw.

1. **What is sampling without replacement? Give example.**
2. Sampling without replacement is a sampling technique in statistics where each element in a population or dataset can be selected only once during the sampling process. Once an element is selected, it is removed from the population, and subsequent draws are made from the reduced set of remaining elements.

**Example** :

Suppose we have a deck of 52 playing cards, and we want to draw three cards from the deck without replacement.

Step 1: We randomly select one card from the deck. Let's say we draw the 7 of hearts. After noting down the card, we remove it from the deck, leaving 51 cards in the deck.

Step 2: We reach into the reduced deck and randomly select another card. This time, we draw the King of spades. We note down the card and remove it from the deck, leaving 50 cards in the deck.

Step 3: We perform the final draw. We reach into the remaining deck and randomly select one last card. This time, we draw the 3 of clubs. We note down the card and remove it from the deck.

After three draws, we have the following sequence of cards: 7 of hearts, King of spades, 3 of clubs. Each card was drawn without replacement, meaning that once a card was selected, it was removed from the deck, and subsequent draws were made from the reduced deck.

In sampling without replacement, the probability of drawing a specific card changes with each draw. For example, the probability of drawing the King of spades in the second draw was 1/51 (since there were 51 cards remaining after the first draw), whereas the probability of drawing the 7 of hearts in the first draw was 1/52 (since all 52 cards were available for the first draw).

1. **What is hypothesis? Give example.**

a hypothesis is a statement or proposition about a population parameter or the relationship between variables that can be tested using data and statistical methods. Hypotheses are formulated to make predictions or draw inferences about the population based on the sample data.

There are two main types of hypotheses in statistical testing:

1. Null Hypothesis (H0): The null hypothesis is a statement of no effect or no difference. It represents the status quo or the assumption that there is no significant relationship or effect between variables. It is often denoted by "H0" and is the hypothesis that researchers aim to test against an alternative hypothesis.

Example of a null hypothesis: H0: There is no difference in the average exam scores between students who received tutoring and students who did not receive tutoring.

1. Alternative Hypothesis (Ha or H1): The alternative hypothesis is a statement that contradicts the null hypothesis and represents the claim or hypothesis researchers want to support with evidence from the data. It proposes a specific effect or relationship between variables.

Example of an alternative hypothesis: Ha: Students who received tutoring have higher average exam scores than students who did not receive tutoring.